Quantum Electromagnetic Field in Time Domain

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The electromagnetic (EM) field is often conveniently first quantized in the frequency domain, as many sources of light have narrow-band characteristics. This is not so, however, for many pulsed sources, including those used for ultrafast photonics, or in the microwave domain, where pulses with very large bandwidth can be generated easily. Measurements of the EM field in those circumstances require a good framework for first quantization in time rather than frequency. Questions then arise, such as: what is the number of photons in a unicycle or subcycle pulse, and can we predict their times of arrival; or what are the equivalent of quadratures in time domain?

We show that there is a fundamental difference between the EM field and its second quantized photonic counterpart: the ladder operators of the latter are not directly proportional to the EM field. The difference is inconspicuous in the frequency domain and is often not mentioned. It becomes very clear and very important in time domain. There is, for instance, a clear difference between the statistics of photons and energy in subcycle pulses. Interestingly, as the energy, or the Hamiltonian, is most directly linked to the number of photons in the frequency domain, equivalently in time domain there is a natural quantity associated with photons: their time of arrival. It is indeed possible to define a weighted time of arrival operator $\hat{\theta}$ which has an interesting commutator with the Hamiltonian [1]

$$\left[\hat{H}, \hat{\theta}\right] = i\hbar \hat{N},$$

(1)

where $\hat{N}$ is the photon number operator. The existence of such an operator puts time and energy on the same footing with respect to the notion of photon, and the latter is clearly cast as a quantum of action, not energy.

Quadratures of the EM field are very well defined when the field is first quantized in the frequency domain. We show that there exists a natural extension of these in time domain, and that quadrature probes are always a Hilbert transform of one another. The Hilbert transform being non-causal, there is a profound relation with the two-state vector formalism of Aharonov, Bergmann, and Lebowitz [2] that requires a forward and a backward-evolving quantum states. In fact, a trace on an oscilloscope or a field measurement performed with electro-optical sampling [3] can be thought of as a two-state vector representation of the EM field and its photonic counterpart for all times between the first and last measurements.

One important feature of the generalized quadratures is their odd/even symmetries, as shown in Fig. 1. For the case of electro-optical sampling, we show that these features are linked to interference between the sum frequency and difference frequency generated signals in the nonlinear crystal. For voltage measurements in the microwave domain, they stem from symmetries of the transforms used to compute them.

Figure 1: Two probe pulse quadratures in frequency (left) and time (right) domain.