Photonic bands in 230 space groups

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The wide-range application of nanophotonics benefits from the enormous design space of three-dimensional sub-wavelength structures. In this work, we study the space group constraints on photonic dispersions for all 230 space groups with time-reversal symmetry. Our theory carefully treats the unique singular point of photonic bands at zero frequency and momentum, which distinguishes photonic bands from their electronic counterpart [1,2]. The results are given in terms of minimal band connectivities at zero (M) and non-zero frequencies (M'). Topological band degeneracies [3] are guaranteed to be found in space groups that do not allow band gaps between the second and third photonic bands (M > 2). Our work provides theoretical guidelines for the choice of spatial symmetries in photonics design.

![Photonic bands diagram](image)

Fig. 1. (a) Two types of band connectivities (M and M') are distinguished by whether they are connected to ω = |k| = 0. (b) M = 4 for space group 4 (P2₁) containing a two-fold screw. (c) M = 2 for space group 7 (Pc) containing a glide. The numbers aside the vertical axes and the colors of the dispersions indicate the eigenvalue of the nonsymmorphic symmetry.

We determine the 104 space groups that allow a band gap of M = 2. They are a translationengleiche-(t-)subgroup of at least one of the 16 key space groups, underscored in the list.


Our results are consistent with the known photonic band gaps between the second and third bands. For example, the three highest space groups of M = 2 are diamond (227), simple cubic (221), and single gyroid (214).

We show that M > 2 for the rest 126 groups, who are t-supergroups of at least one these 22 key space groups:


Our results are consistent with the band structures of known photonic crystals in which no band gaps are found between the second and third bands, such as the hexagonal close packing (194), tetrahedral (224), face-centered-cubic (225), body-centered cubic (229), and double-gyroid (230) dielectric photonic crystals.

References